Implicit Large Eddy Simulation (ILES) for High Reynolds Number Flows

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Implicit Large Eddy Simulation

Outline:

- **What is $ILES$?**
- **What are its advantages?**
- **Historical perspective**
- **Why does it work?**
- **Some examples**

What is $ILES$

$ILES$ **is the direct application of a fluid solver to a high Reynolds number fluid flow with no explicit turbulence model.**

- **The truncation terms of the algorithm serve as an effective model of the effects of the unresolved scales.**

- **Fluid solvers based on Nonoscillatory Finite Volume (NFV) approximations work effectively for** $ILES$**.**

- **Fluid solvers based on pseudospectral methods, leapfrog methods, advective form methods, etc. do not work for** $ILES$**.**

$\Rightarrow$ $ILES$ **appears to be a unique property of NFV methods.**

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Some Advantages of $ILES$

- **Computationally efficient**
- **Easy to implement**
- **Not necessary to know if the flow is turbulent;** ***the same solver can be used for all flows***
- **NFV methods are** ***adaptive***
- **NFV methods have** ***no parameters***

Early History of $ILES$

Boris, Oran, Grinstein, 1992: used FCT to model combusting flows.

Linden, Redondo, Youngs, 1994: used van Leer schemes to model fluid instabilities.

Porter, Pouquet, Woodward, 1994: used PPM to model astrophysical jets (highly compressible flow)

Margolin, Smolarkiewicz, Sorbjan, 1999: used MPDATA to model atmospheric boundary layers and global climate

Earliest description of $ILES$ by Boris (1988)

MPDATA Experience

MPDATA (Smolarkiewicz & Margolin, 1998) is an NFV method based on iterated upwinding. It is not monotonicity preserving. Some MPDATA examples of $ILES$ application areas include:

- Atmospheric boundary layers; oceans; climate
- Idealized turbulence decay
- Solar convection
- Flows with strong shocks
- Fluid instabilities

Some Underlying Ideas

Some basic ideas that underlie the $ILES$ approach

- von Neumann & Richtmyer, 1951– artificial viscosity
- Smagorinsky, 1963 – subgrid scale models
- Hirt, 1969 – truncation terms vs. subgrid scale models
- Belotserkovskii, 1986 – flux form and computational stability
- Merriam, 1987 – monotonicity and the second law

A Rationale for $ILES$

Our thesis can be succinctly stated as follows:

The success of $ILES$ *follows from the fact that NFV methods accurately solve the equations that describe the dynamics of finite volumes of fluid.*

These equations differ from the Navier-Stokes PDEs, and explicitly contain information about the volume over which one averages.

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Volume Averaged Velocities – A Specific Example

We will define the volume-averaged velocities

$$\bar{u}(x,y) \equiv \frac{1}{\Delta x\,\Delta y} \int_{x-\frac{1}{2}\,\Delta x}^{x+\frac{1}{2}\,\Delta x} \int_{y-\frac{1}{2}\,\Delta y}^{y+\frac{1}{2}\,\Delta y} u(x',y')\,dx'\,dy' \qquad (1)$$

and

$$\bar{v}(x,y) \equiv \frac{1}{\Delta x\,\Delta y} \int_{x-\frac{1}{2}\,\Delta x}^{x+\frac{1}{2}\,\Delta x} \int_{y-\frac{1}{2}\,\Delta y}^{y+\frac{1}{2}\,\Delta y} v(x',y')\,dx'\,dy' \qquad (2)$$

That is, here we have chosen a specific volume of integration, a rectangle, that mimics a computational cell in a regular mesh.

Finite Scale Navier-Stokes Equations in 2D

The final result for the finite-scale (volume-averaged) momentum equations, to $\mathcal{O}(\Delta x^2, \Delta y^2)$ is:

$$\begin{aligned}\frac{\partial \bar{u}}{\partial t} = & - (\bar{u}^2)_x - (\bar{v}\bar{u})_y - \bar{P}_x + \nu \left(\bar{u}_{xx} + \bar{u}_{yy}\right) \\ & - \frac{1}{3}\left(\frac{\Delta x}{2}\right)^2 \left[(\bar{u}_x\bar{u}_x)_x + (\bar{v}_x\bar{u}_x)_y\right] - \frac{1}{3}\left(\frac{\Delta y}{2}\right)^2 \left[(\bar{u}_y\bar{u}_y)_x + (\bar{u}_y\bar{v}_y)_y\right]\end{aligned} \qquad (3)$$

$$\begin{aligned}\frac{\partial \bar{v}}{\partial t} = & - (\bar{u}\bar{v})_x - (\bar{v}^2)_y - \bar{P}_y + \nu \left(\bar{v}_{xx} + \bar{v}_{yy}\right) \\ & - \frac{1}{3}\left(\frac{\Delta x}{2}\right)^2 \left[(\bar{u}_y\bar{u}_y)_x + (\bar{u}_y\bar{v}_y)_y\right] - \frac{1}{3}\left(\frac{\Delta y}{2}\right)^2 \left[(\bar{u}_y\bar{v}_y)_x + (\bar{v}_y\bar{v}_y)_y\right]\end{aligned} \qquad (4)$$

Inviscid Energy Dissipation

$$\frac{dE_{FS}}{dt} = \frac{1}{6}\left(\frac{\Delta x}{2}\right)^2 \langle \bar{u}_x^3 \rangle + \frac{1}{6}\left(\frac{\Delta y}{2}\right)^2 \langle \bar{v}_y^3 \rangle + \frac{1}{6}\left[\left(\frac{\Delta x}{2}\right)^2 - \left(\frac{\Delta y}{2}\right)^2\right] \langle \bar{u}_x \bar{u}_y \bar{v}_x \rangle$$

$$\begin{aligned}
\frac{dE_{ME}}{dt} &= \frac{1}{2}\left(\frac{\Delta x}{2}\right)^2 \left[\frac{1}{3}\langle \bar{u}_x^3 \rangle - \langle |\bar{u}_x^3| \rangle + \frac{1}{3}\langle \bar{u}_x \bar{v}_x^2 \rangle - \langle |\bar{u}_x| \bar{v}_x^2 \rangle\right] \\
&+ \frac{1}{2}\left(\frac{\Delta y}{2}\right)^2 \left[\frac{1}{3}\langle \bar{v}_y^3 \rangle - \langle |\bar{v}_y^3| \rangle + \frac{1}{3}\langle \bar{v}_y \bar{u}_y^2 \rangle - \langle |\bar{v}_y| \bar{u}_y^2 \rangle\right] \\
&- \frac{1}{3}\left(\frac{\Delta x}{2}\right)^2 \left[\langle \bar{v} \bar{v}_x \bar{u}_{xx} \rangle - \langle \bar{u} \bar{v}_x \bar{v}_{xx} \rangle\right] - \frac{1}{3}\left(\frac{\Delta y}{2}\right)^2 \left[\langle \bar{u} \bar{u}_y \bar{v}_{yy} \rangle - \langle \bar{v} \bar{u}_y \bar{u}_{yy} \rangle\right]
\end{aligned}$$

Discussion Points

- The dynamics of finite volumes of fluid is governed by different equations than Navier-Stokes; additional terms appear that depend on the scales of the volumes.

- The truncation analysis of the discrete equations of NFV algorithms contain similar terms.

- The finite-scale equations *do not* depend on the details of the unresolved scales. This implies that the small scales are enslaved by the larger scales.

A fuller comparison and discussion is included in the accompanying paper.

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Some Examples

3D simulations of decaying turbulence using MPDATA (Smolarkiewicz & Margolin, 1998).

resolution 256^3

Initial setup in Herring & Kerr (1993).

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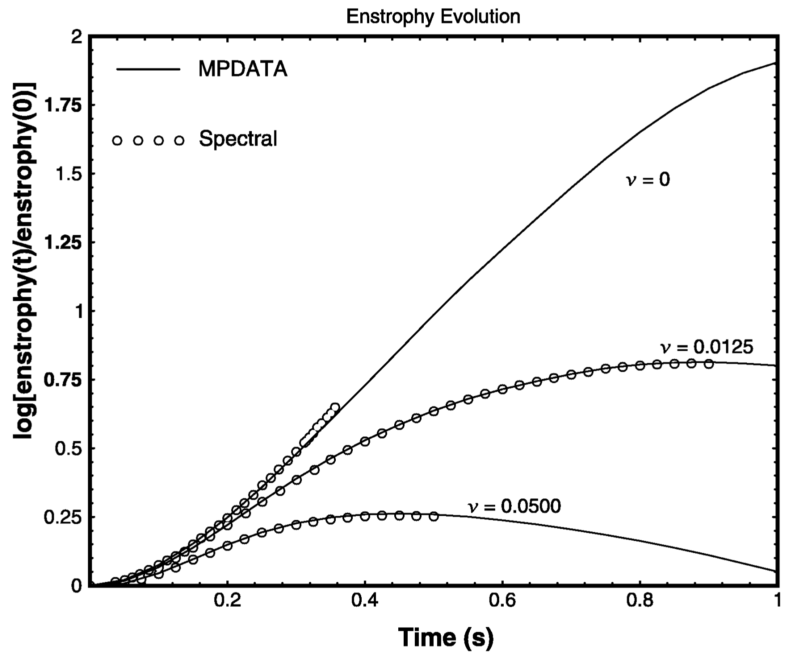
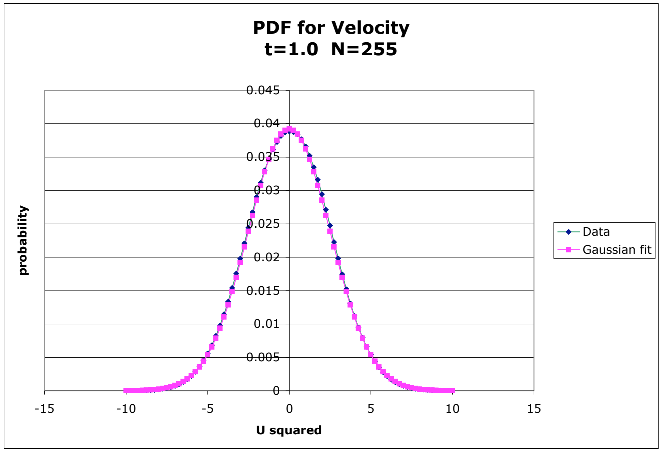
Comparisons with Pseudospectral Simulations



Time evolution of global enstrophy for three viscosities

Probability Distribution Function (PDF) of Velocity



Shows a Gaussian distribution, whose ”temperature” is the global kinetic energy.

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Summary of Results

- **Velocity PDFs are Maxwellian (Gaussian)**
- **Longitudinal velocity increments are skewed**
- **Verified Kolmogorov's $\frac{4}{5}$ law**
- **Verified scaling of spatial moments**
- **Demonstrated adaptivity of methods to explicit viscosity**

Simulating **turbulence may be easier than** ***understanding*** **turbulence.**

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